UNDERSTANDING LEVERS

By Mitch Ricketts

Math Toolbox is designed to help readers apply STEM principles to everyday safety issues. Many readers may feel apprehensive about math and science. This series employs various communication strategies to make the learning process easier and more accessible.

This Math Toolbox article is the second of two installments on the role of levers in occupational safety. In the first installment ("The Case of the Tipping Forklift," *PSJ* September 2020, pp. 45-51), we considered the hazards associated with out-of-control levers in workplaces. This article will delve more deeply to understand the three main classes of levers, the concept of mechanical advantage and some associated calculations.

Classes of Levers

Figure 1 depicts two examples of first-class levers: a stylized lever at top and a pry bar as an everyday example at bottom. Remember from the first installment that a lever is a rigid device that pivots on a fulcrum. The fulcrum is the pivot point, or the point about which the lever rotates. The lever has two "arms": The load arm (or output arm) is the portion of the lever directly connected to the load. The effort arm (or arm of applied force) is the portion of the lever to which we apply the effort, or input force. Force is a push or pull in a particular direction (e.g., up, down, left, right, rotational). The magnitude (or amount) of force is often specified in units of weight, such as ounces, pounds or newtons. In a first-class lever, the effort arm and load arm are located on opposite sides of the fulcrum, and effort is applied in the same direction as the force exerted by the load.

In a second-class lever (Figure 2), the effort arm and load arm are located on the same side of the fulcrum. Effort is applied in the opposite direction, compared with the force exerted by the load. Furthermore, the load lies between the point of effort and the fulcrum. A wheelbarrow is an everyday example of a second-class lever.

In a third-class lever (Figure 3), the effort arm and load arm are located on the same side of the fulcrum. The direction of effort is opposite the direction of the force exerted by the load. Uniquely for third-class levers, the point of effort lies between the load and the fulcrum. Since these levers tend to be inefficient, they are less common in everyday technology. Third-class levers are commonplace in anatomy, however, as illustrated by the human elbow, where the bicep exerts upward effort, the elbow serves as the fulcrum, and the tendon attachment denotes the line of applied effort on the forearm.

Recall from the first installment on levers that the rotational force of a pivoting lever is known as torque (τ). Torque is stated in units of distance times weight, such as foot-pounds (ft-lb) and newton-meters (N-m). The direction of torque may be denoted with a positive sign for clockwise motion and a negative sign for counterclockwise rotation. As in the first installment, we will simplify by using the directionless, absolute value of torque, designated as $|\tau|$.

Calculating Torque for Each Lever Class

In the first installment, we applied the formula for torque (absolute value) to first-class levers as follows:

$$|\tau| = f \cdot d$$

FIGURE 1
FIRST-CLASS LEVER EXAMPLES

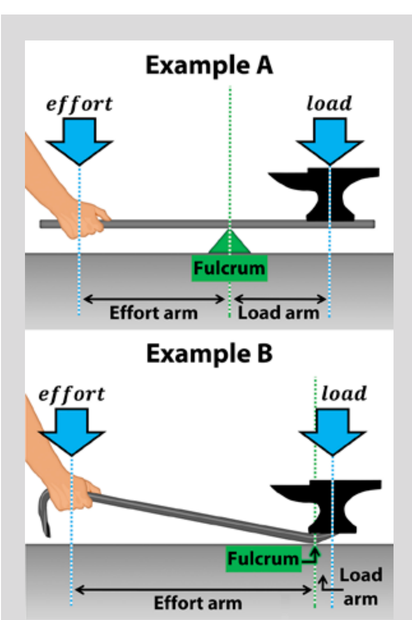


FIGURE 2
SECOND-CLASS LEVER EXAMPLES

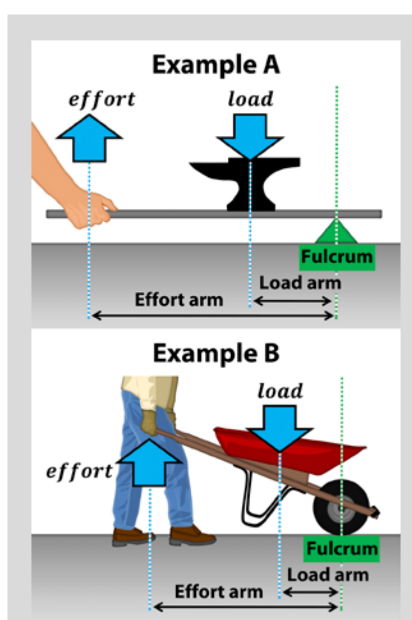


FIGURE 3
THIRD-CLASS LEVER EXAMPLES

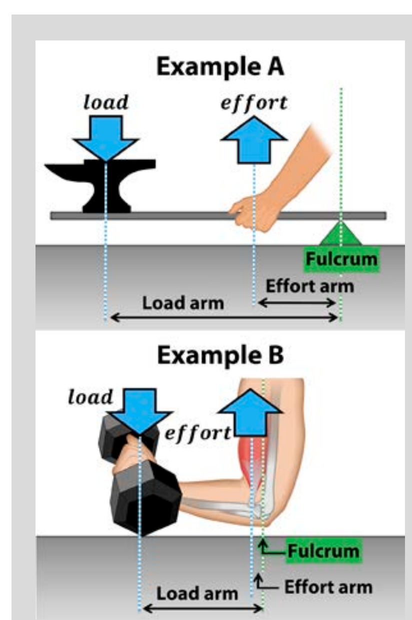


FIGURE 4
OPPOSING TORQUES OF EFFORT & LOAD IN A FIRST-CLASS LEVER

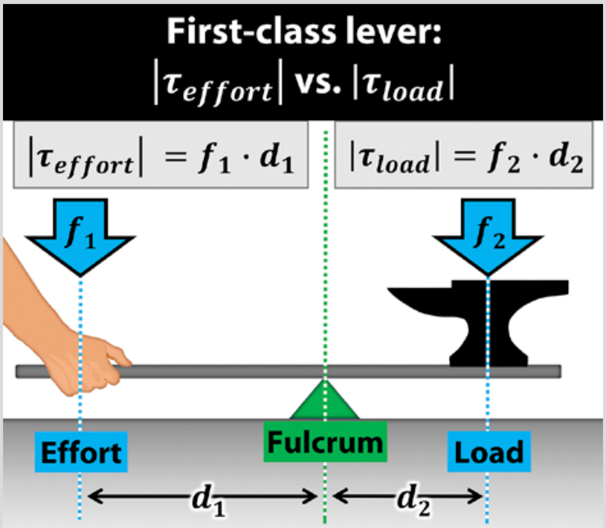
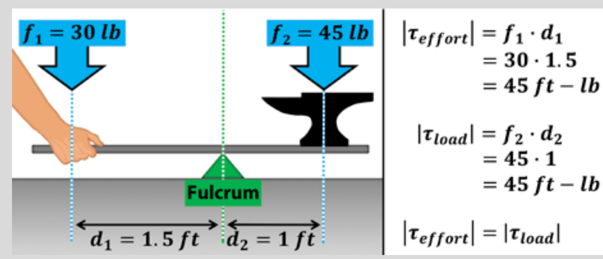


FIGURE 5
EXAMPLE: BALANCED FIRST-CLASS LEVER

Example problem. Equivalent torques result in a balanced first-class lever. In this case, a 30-lb force applied to an effort arm of 1.5 ft balances the torque created by a 45-lb force applied to a load arm of 1 ft.



where:
$|\tau|$ = torque (absolute value); a turning or twisting force
f = force applied perpendicularly (at a right angle) to the lever arm
d = distance at which the force is applied; the distance from the fulcrum to the line of applied force (measured perpendicularly to the line of applied force)

Note: When force is not applied perpendicularly to the lever arm, a more general formula is used to account for the angle of applied force: $\tau = f \cdot d \cdot \sin\theta$.

Continuing our review, we can calculate torque for both sides of a first-class lever as shown in Figure 4. For the effort arm, torque may be designated as $|\tau_{effort}|$. Force and distance in the effort arm may be designated as f_1 and d_1, respectively. For the load arm, torque may be designated as $|\tau_{load}|$, with force and distance in the load arm denoted as f_2 and d_2. If the opposing torques are equal, then $|\tau_{effort}| = |\tau_{load}|$ and the lever will be balanced. On the other hand, if torque created by the effort arm is greater than torque created by the load arm, then $|\tau_{effort}| > |\tau_{load}|$ and the load will rise. Finally, if torque created by the effort arm is less than torque created by the load arm, then $|\tau_{effort}| < |\tau_{load}|$ and the load will descend.

First-class lever calculated example: Imagine you have used a first-class lever to raise a load consisting of a 45-lb weight, as illustrated in Figure 5. The load's center of gravity is located 1 ft to the right of the fulcrum. (Center of gravity is the average location of a body's weight, or the location where a body's total weight is assumed to be concentrated.)

To keep the load balanced (so it moves neither up nor down), you exert a downward force of 30 lb with your hand. We will consider this 30-lb force to be applied at a point that represents the average location where your hand presses on the lever, which in this case is a distance of 1.5 ft to the left of the fulcrum. We will assume the weights of the lever arms are negligible (or, alternatively, that the weights of the lever arms are balanced on both sides of the fulcrum). We will also assume friction is negligible. Under these conditions, how many foot-pounds of torque are created by the effort arm to balance the torque created by the load arm?

We calculate torque created by the effort arm ($|\tau_{effort}|$) based on the following data:

•Your hand applies a downward force of 30 lb perpendicularly to the effort arm. This is the value of f_1 in the formula.

•Your hand applies this force at an average distance of 1.5 ft from the fulcrum. This is the value of d_1 in the formula.

Based on these data, we calculate torque generated by the effort arm ($|\tau_{effort}|$) in ft-lb as follows:

Step 1: Start with the equation for torque of the effort arm:

$$|\tau_{effort}| = f_1 \cdot d_1$$

Step 2: Insert the known values for applied force (f_1 = 30 lb) and distance (d_1 = 1.5 ft). Then solve for $|\tau_{effort}|$ in ft-lb:

$$|\tau_{effort}| = f_1 \cdot d_1 = 30 \cdot 1.5 = 45\ ft - lb$$

Step 3: Our calculation indicates that the effort arm is generating a torque of 45 ft-lb to balance the torque created by the load arm.

We will now calculate torque created by the load arm ($|\tau_{load}|$) to confirm that the torques are balanced, based on the following data:

•The weight applies a downward force of 45 lb perpendicularly to the load arm. This is the value of f_2 in the formula.

•The weight applies this force at an average distance of 1 ft from the fulcrum. This is the value of d_2 in the formula.

Based on these data, we calculate torque generated by the load arm ($|\tau_{load}|$) in ft-lb as follows:

Step 1: Start with the equation for torque of the load arm:

$$|\tau_{load}| = f_2 \cdot d_2$$

Step 2: Insert the known values for weight of the load (f_2 = 45 lb) and distance (d_2 = 1 ft). Then solve for $|\tau_{load}|$ in ft-lb:

$$|\tau_{load}| = f_2 \cdot d_2 = 45 \cdot 1 = 45\ ft - lb$$

Step 3: Our calculation indicates that the load arm is generating a torque of 45 ft-lb, which is equal to the torque of the effort arm, meaning the torques are balanced. In contrast, if the torques had been unbalanced, the load would either rise (if $|\tau_{effort}| > |\tau_{load}|$) or descend (if $|\tau_{effort}| < |\tau_{load}|$).

Second-class lever calculated example: We calculate torque for a second-class lever using the same formula. However, note that the effort and load arms overlap because they are located on the same side of the fulcrum, as shown in Figure 6 (p. 54). Also note that in second-class levers the effort is applied in a direction opposite to the direction of the force of the load. The value of d_1 will be the entire distance from the fulcrum to the average point of applied effort. The value of d_2 will be the distance from the load's center of gravity to the fulcrum. Once again, the opposing torques are designated as $|\tau_{effort}|$ and $|\tau_{load}|$. Similarly, the effort and load forces designated as f_1 and f_2, and the distances are designated as d_1 and d_2 for the effort and load arms respectively. As always, $|\tau_{effort}|=|\tau_{load}|$ when the lever is balanced; the load will rise when $|\tau_{effort}| > |\tau_{load}|$ and the load will descend when $|\tau_{effort}| < |\tau_{load}|$.

To calculate opposing torques in a second-class lever, imagine you have used the lever shown in Figure 7 to raise a load consisting of a 40-lb weight. The load's center of gravity is located 0.75 ft to the left of the fulcrum. To keep the load balanced, you exert an upward force of 15 lb with your hand. Your hand exerts this force at an average distance of 2 ft to the left of the fulcrum. Assuming friction and the weights of the lever arms are negligible, how many foot-pounds of torque are created by the effort arm to balance the torque created by the load arm?

We calculate the torque created by the effort arm ($|\tau_{effort}|$) based on the following data:

•Your hand applies an upward force of 15 lb perpendicularly to the effort arm. This is the value of f_1 in the formula.

•Your hand applies this force at an average distance of 2 ft from the fulcrum. This is the value of d_1 in the formula.

Based on these data, we calculate torque generated by the effort arm ($|\tau_{effort}|$) in ft-lb as follows:

Step 1: Start with the equation for torque of the effort arm:

$$|\tau_{effort}| = f_1 \cdot d_1$$

Step 2: Insert the known values for applied force (f_1 = 15 lb) and distance (d_1 = 2 ft). Then solve for $|\tau_{effort}|$ in ft-lb:

$$|\tau_{effort}| = f_1 \cdot d_1 = 15 \cdot 2 = 30\ ft - lb$$

Step 3: Our calculation indicates the effort arm is generating a torque

FIGURE 6

OPPOSING TORQUES OF EFFORT & LOAD IN A SECOND-CLASS LEVER

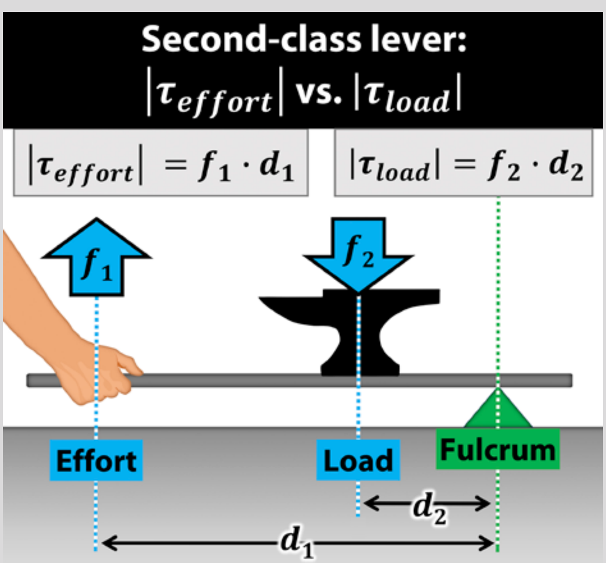


FIGURE 8

OPPOSING TORQUES OF EFFORT & LOAD IN A THIRD-CLASS LEVER

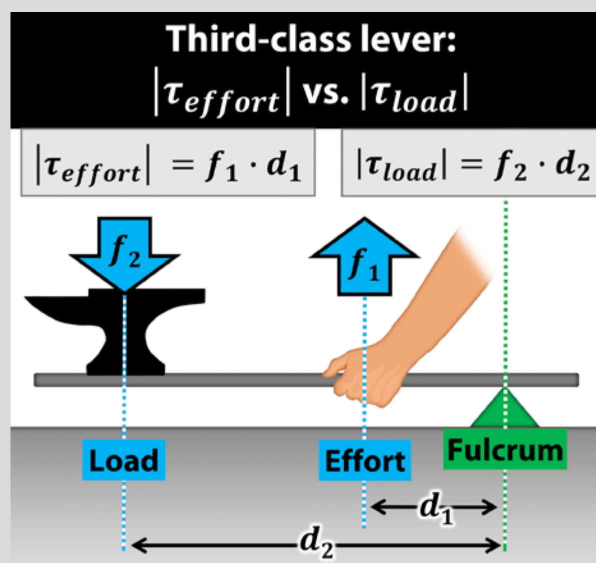
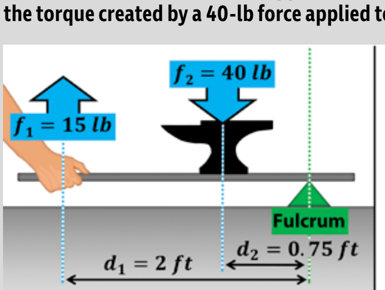


FIGURE 7

EXAMPLE: BALANCED SECOND-CLASS LEVER

Example problem. Equivalent torques result in a balanced second-class lever. In this case, a 15-lb force applied to an effort arm of 2 ft balances the torque created by a 40-lb force applied to a load arm of 0.75 ft.



$$\begin{aligned}|\tau_{effort}| &= f_1 \cdot d_1\\ &= 15 \cdot 2\\ &= 30\ ft - lb\end{aligned}$$

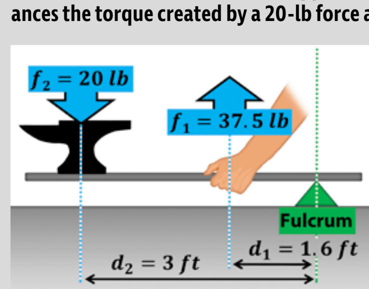
$$\begin{aligned}|\tau_{load}| &= f_2 \cdot d_2\\ &= 40 \cdot 0.75\\ &= 30\ ft - lb\end{aligned}$$

$$|\tau_{effort}| = |\tau_{load}|$$

FIGURE 9

EXAMPLE: BALANCED THIRD-CLASS LEVER

Example problem. Equivalent torques result in a balanced third-class lever. In this case, a 37.5-lb force applied to an effort arm of 1.6 ft balances the torque created by a 20-lb force applied to a load arm of 3 ft.



$$\begin{aligned}|\tau_{effort}| &= f_1 \cdot d_1\\ &= 37.5 \cdot 1.6\\ &= 60\ ft - lb\end{aligned}$$

$$\begin{aligned}|\tau_{load}| &= f_2 \cdot d_2\\ &= 20 \cdot 3\\ &= 60\ ft - lb\end{aligned}$$

$$|\tau_{effort}| = |\tau_{load}|$$

of 30 ft-lb to balance the torque of the load arm.

We will now calculate torque created by the load arm ($|\tau_{load}|$) to confirm that the torques are balanced. Our data are as follows:

•The weight applies a downward force of 40 lb perpendicularly to the load arm. This is the value of f_2 in the formula.

•The weight applies this force at an average distance of 0.75 ft from the fulcrum. This is the value of d_2 in the formula.

Based on these data, we calculate torque generated by the load arm ($|\tau_{load}|$) in ft-lb as follows:

Step 1: Start with the equation for torque of the load arm:

$$|\tau_{load}| = f_2 \cdot d_2$$

Step 2: Insert the known values for weight of the load (f_2 = 40 lb) and distance (d_2 = 0.75 ft). Then solve for $|\tau_{load}|$ in ft-lb:

$$|\tau_{load}| = f_2 \cdot d_2 = 40 \cdot 0.75 = 30\ ft - lb$$

Step 3: Our calculation indicates that the load arm is generating a torque of 30 ft-lb, which is equal to the torque of the effort arm, meaning the torques are balanced. In contrast, if the torques had been unbalanced, the load would either rise (if $|\tau_{effort}| > |\tau_{load}|$) or descend (if $|\tau_{effort}| < |\tau_{load}|$).

Third-class lever calculated example: We use the same formula to calculate torque for a third-class lever. The effort and load arms overlap on the same side of the fulcrum (as for second-class levers), but in third-class levers the effort arm is always shorter than the load arm, as shown in Figure 8.

To calculate opposing torques in a third-class lever, imagine you have used the lever shown in Figure 9 to raise a load consisting of a 20-lb weight. The load's center of gravity is located 3 ft to the left of the fulcrum. To keep the load balanced, you exert an upward force of 37.5 lb with your hand. Your hand exerts this force at an average distance of 1.6 ft to the left of the fulcrum. Assuming friction and the weights of the lever arms are negligible, how many foot-pounds of torque are created by the effort arm to balance the torque created by the load arm?

We calculate torque created by the effort arm ($|\tau_{effort}|$) based on the following data:

•Your hand applies an upward force of 37.5 lb perpendicularly to the effort arm. This is the value of f_1 in the formula.

•Your hand applies this force at an average distance of 1.6 ft from the fulcrum. This is the value of d_1 in the formula.

Based on these data, can calculate torque generated by the effort arm ($|\tau_{effort}|$) in ft-lb as follows:

Step 1: Start with the equation for torque of the effort arm:

$$|\tau_{effort}| = f_1 \cdot d_1$$

Step 2: Insert the known values for applied force (f_1 = 37.5 lb) and distance (d_1 = 1.6 ft). Then solve for $|\tau_{effort}|$ in ft-lb:

$$|\tau_{effort}| = f_1 \cdot d_1 = 37.5 \cdot 1.6 = 60\ ft - lb$$

Step 3: Our calculation indicates that the effort arm is generating a torque of 60 ft-lb to balance the torque of the load arm.

We will now calculate torque created by the load arm ($|\tau_{load}|$) to confirm that the torques are balanced. Our data are as follows:

•The weight applies a downward force of 20 lb perpendicularly to the load arm. This is the value of f_2 in the formula.

•The weight applies this force at an average distance of 3 ft from the fulcrum. This is the value of d_2 in the formula.

Based on these data, we calculate torque generated by the load arm ($|\tau_{load}|$) in ft-lb as follows:

Step 1: Start with the equation for torque of the load arm:

$$|\tau_{load}| = f_2 \cdot d_2$$

Step 2: Insert the known values for weight of the load (f_2 = 20 lb) and distance (d_2 = 3 ft). Then solve for $|\tau_{load}|$ in ft-lb:

$$|\tau_{load}| = f_2 \cdot d_2 = 20 \cdot 3 = 60\ ft - lb$$

Step 3: Our calculation indicates the load arm is generating a torque of 60 ft-lb, which is equal to the torque of the effort arm. Note that the applied effort (37.5 lb) is greater than the weight of the load (20 lb) because the effort arm is shorter than the load arm.

You Do the Math

Apply your knowledge to the following questions. Answers are on p. 63.

1. Imagine you have used a first-class lever to raise a load consisting of a 100-lb weight, as illustrated in Figure 10. The load's center of gravity is located 2 ft to the right of the fulcrum. To keep the load balanced (so it moves neither up nor down), you exert a downward force of 50 lb with your hand. Your hand exerts this force at an average distance of 4 ft to the left of the fulcrum. Answer the following questions, based on the assumption that friction and the weights of the lever arms are negligible:

a. How many foot-pounds of torque are created by the effort arm to balance the torque created by the load arm? Use the equation for torque of the effort arm ($|\tau_{effort}|$) and solve in units of foot-pounds (ft-lb).

b. How many foot-pounds of torque are created by the load arm? Use the equation for torque of the load arm ($|\tau_{load}|$) and solve in units of foot-pounds (ft-lb).

FIGURE 10
YOU DO THE MATH, PROBLEMS 1 & 4

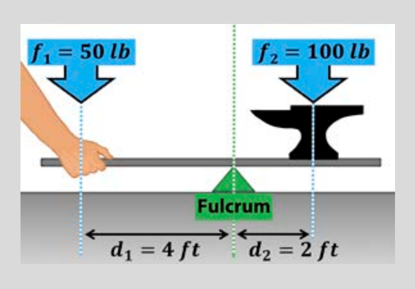


FIGURE 11
YOU DO THE MATH, PROBLEMS 2 & 5

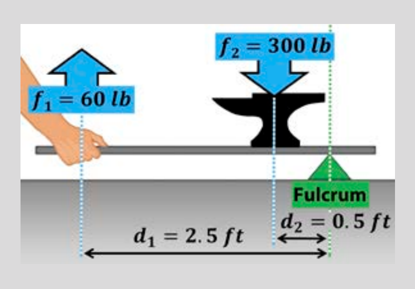
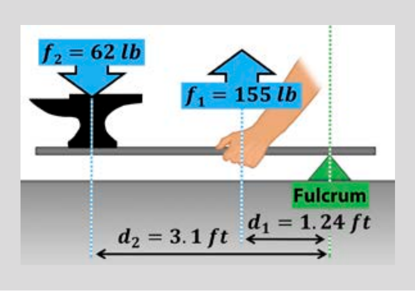


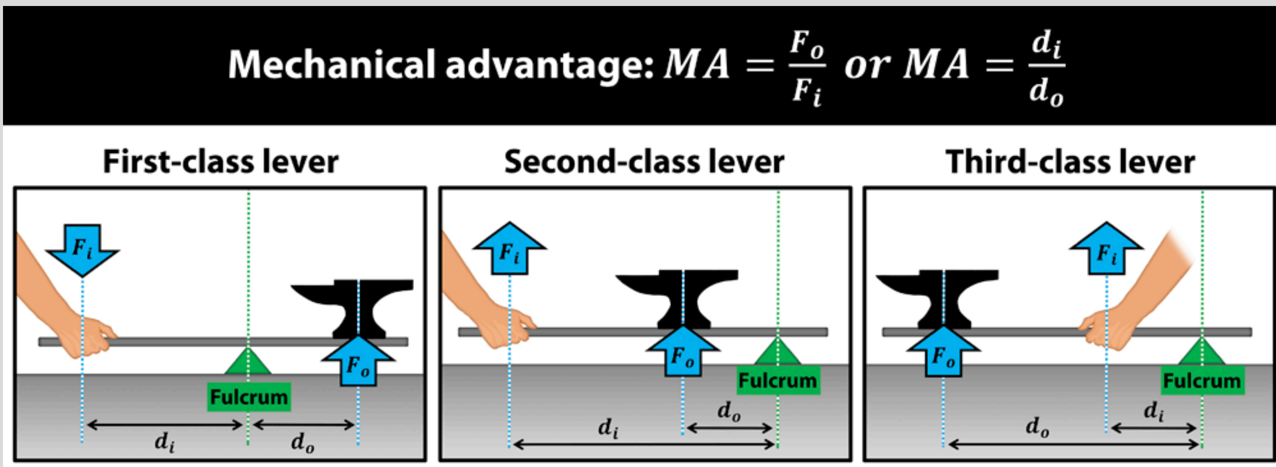
FIGURE 12
YOU DO THE MATH, PROBLEMS 3 & 6



2. Imagine you have used a second-class lever to raise a load consisting of a 300-lb weight, as illustrated in Figure 11. The load's center of gravity is located 0.5 ft to the left of the fulcrum. To keep the load balanced (so it moves neither up nor down), you exert an upward force of 60 lb with your hand. Your hand exerts this force at an average distance of 2.5 ft to the left of the fulcrum. Answer the following questions, based on the assumption

FIGURE 13
MECHANICAL ADVANTAGE

Mechanical advantage: Variables F_i, F_o, d_i and d_o for three classes of levers. For balanced levers (and when friction and the weights of the lever arms are negligible), the magnitudes of F_i and F_o are equal to the magnitudes of f_1 and f_2 from the earlier equations; however, the direction of F_o is opposite the direction of the previous f_2. The values of d_i and d_o are equivalent to d_1 and d_2 from the previous equations. For unbalanced levers, the magnitude of F_o must either be measured with a scale or calculated as $F_o = |\tau_{effort}| \div d_o$ or as $F_o = MA \cdot F_i$.



that friction and the weights of the lever arms are negligible:

a. How many foot-pounds of torque are created by the effort arm to balance the torque created by the load arm? Use the equation for torque of the effort arm ($|\tau_{effort}|$) and solve in units of foot-pounds (ft-lb).

b. How many foot-pounds of torque are created by the load arm? Use the equation for torque of the load arm ($|\tau_{load}|$) and solve in units of foot-pounds (ft-lb).

3. Imagine you have used a third-class lever to raise a load consisting of a 62-lb weight, as illustrated in Figure 12 (p. 55). The load's center of gravity is located 3.1 ft to the left of the fulcrum. To keep the load balanced (so it moves neither up nor down), you exert an upward force of 155 lb with your hand. Your hand exerts this force at an average distance of 1.24 ft to the left of the fulcrum. Answer the following questions, based on the assumption that friction and the weights of the lever arms are negligible:

a. How many foot-pounds of torque are created by the effort arm to balance the torque created by the load arm? Use the equation for torque of the effort arm ($|\tau_{effort}|$) and solve in units of foot-pounds (ft-lb).

b. How many foot-pounds of torque are created by the load arm? Use the equation for torque of the load arm ($|\tau_{load}|$) and solve in units of foot-pounds (ft-lb).

Mechanical Advantage

Mechanical advantage (MA) is the amplification of force by a machine. Higher values of mechanical advantage translate to greater leverage. Specifically, the value of MA will be greater than one (MA > 1) when a machine's output force is greater than the input force. The value of MA will be equal to one (MA = 1) when a machine's output force equals the input force. The value of MA will be less than one (MA < 1) when a machine's output force is less than the input force. For example, if MA = 4, the machine's output force is four times the input force; if MA = 2, the output force is two times the input force; if MA = 0.75, the output force is three-quarters the input force; if MA = 0.25, the output force is one-quarter the input force, and so forth.

For levers, mechanical advantage can be calculated in either of two ways: 1. as output force (F_o) divided by input force (F_i); or 2. as the length of the input arm (d_i) divided by the length of the output arm (d_o). When the lever is balanced and when friction and the weights of the lever arms are negligible, F_i and F_o are equivalent to f_1 and f_2 from the previous equations. In other words, the input force (F_1) is simply the effort we apply to the lever and the output force (F_2) has a magnitude equal to the weight of the load (but a direction opposite that of the weight of the load). Similarly, d_i and d_o are equivalent to d_1 and d_2 from the previous equations. The two equations are illustrated in Figure 13 and are defined as follows:

$$MA = \frac{F_o}{F_i} \text{ or } MA = \frac{d_i}{d_o}$$

where:

MA = mechanical advantage, or amplification of force by a lever (ignoring friction, flexure and weight of lever components)

F_i = input force; the force applied to the effort arm; equivalent to f_1 in the previous calculations, assuming that friction and the weights of the lever arms are negligible

F_o = output force; the force output through the load arm; for a lever in a state of balance, and when friction and the weights of the lever arms are negligible, F_o is equivalent in magnitude to f_2 (but opposite in direction); for an unbalanced lever, the magnitude of F_o must either be measured with a scale or calculated as $F_o = |\tau_{effort}| \div d_o$ or as $F_o = MA \cdot F_i$

d_i = input distance; the distance from the fulcrum to the point where input force is applied through the effort arm; equivalent to d_1

d_o = output distance; the distance from the fulcrum to the point where output force is applied through the load arm; equivalent to d_2

First-class lever calculated example: Consider the first-class lever illustrated in Figure 5 (p. 53). The lever is balanced. In other words, the lever is moving neither up nor down because the torques are equivalent on both sides of the fulcrum. For balanced levers (assuming friction and the weight of the lever arms are negligible), the magnitudes of F_i and F_o are equal to the magnitudes of f_1 and f_2, respectively. Thus, F_i = 30 lb and F_o = 45 lb.

Based on these data, we can calculate the mechanical advantage (MA) based on the forces F_i and F_o as follows:

Step 1: Start with the equation for mechanical advantage based on force:

$$MA = \frac{F_o}{F_i}$$

Step 2: Insert the known values for the input and output forces ($F_i = 30$ lb; $F_o = 45$ lb). Then solve for MA:

$$MA = \frac{F_o}{F_i} = \frac{45}{30} = 1.5$$

Step 3: Our calculation indicates that the lever has a mechanical advantage of 1.5. In other words, the lever amplifies the input force by a multiple of 1.5 (i.e., every 1 lb of force applied to the input, or effort, arm produces an output of 1.5 lb).

Next, we will calculate mechanical advantage of the lever in Figure 5 (p. 53) based on the distances, d_i and d_o. Since d_i and d_o are equivalent to d_1 and d_2 from the previous equations, $d_i = 1.5$ ft and $d_o = 1$ ft for the lever in Figure 5.

Step 1: Start with the equation for mechanical advantage based on distance:

$$MA = \frac{d_i}{d_o}$$

Step 2: Insert the known values for the input and output distances ($d_i = 1.5$ ft; $d_o = 1$ ft). Then solve for MA:

$$MA = \frac{d_i}{d_o} = \frac{1.5}{1} = 1.5$$

Step 3: Our calculation indicates that the lever has a mechanical advantage of 1.5, confirming the result we obtained using the equation based on force.

Second-class lever calculated example: Consider the second-class lever shown in Figure 7 (p. 54). Since the lever is balanced, the magnitudes of F_i and F_o equal the magnitudes of f_1 and f_2, respectively. In other words, $F_i = 15$ lb and $F_o = 40$ lb.

Based on these data, we can calculate the mechanical advantage (MA) based on the forces as follows:

Step 1: Start with the equation for mechanical advantage based on force:

$$MA = \frac{F_o}{F_i}$$

Step 2: Insert the known values for the input and output forces ($F_i = 15$ lb; $F_o = 40$ lb). Then solve for MA:

$$MA = \frac{F_o}{F_i} = \frac{40}{15} = 2.67$$

(rounded two places beyond the decimal)

Step 3: Our calculation indicates that the lever has a mechanical advantage of 2.67. In other words, the lever amplifies the input force by a multiple of 2.67.

Next, we will calculate mechanical advantage of the lever in Figure 7 (p. 54) based on the distances. Since d_i and d_o are equivalent to d_1 and d_2, we see from Figure 7 that $d_i = 2$ ft and $d_o = 0.75$ ft.

Step 1: Start with the equation for mechanical advantage based on distance:

$$MA = \frac{d_i}{d_o}$$

Step 2: Insert the known values for the input and output distances ($d_i = 2$ ft; $d_o = 0.75$ ft). Then solve for MA:

$$MA = \frac{d_i}{d_o} = \frac{2}{0.75} = 2.67$$

(rounded)

Step 3: Our calculation indicates that the lever has a mechanical advantage of 2.67, confirming the result we obtained using the equation based on force.

Third-class lever calculated example: Consider the third-class lever shown in Figure 9 (p. 54). Since the lever is balanced, the magnitudes of F_i and F_o equal the magnitudes of f_1 and f_2, respectively. In other words, $F_i = 37.5$ lb and $F_o = 20$ lb.

Based on these data, we can calculate the mechanical advantage (MA) based on the forces as follows:

Step 1: Start with the equation for mechanical advantage based on force:

$$MA = \frac{F_o}{F_i}$$

Step 2: Insert the known values for the input and output forces ($F_i = 37.5$ lb; $F_o = 20$ lb). Then solve for MA:

$$MA = \frac{F_o}{F_i} = \frac{20}{37.5} = 0.53$$

(rounded)

Step 3: Our calculation indicates that the lever has a mechanical advantage of 0.53. In other words, the lever amplifies the input force by a multiple of 0.53. Note that since MA is less than one, the output force is less than the input force.

Next, we will calculate mechanical advantage of the lever in Figure 9 (p. 54) based on the distances. Since d_i and d_o are equivalent to d_1 and d_2, we see from Figure 9 that $d_i = 1.6$ ft and $d_o = 3$ ft.

Step 1: Start with the equation for mechanical advantage based on distance:

$$MA = \frac{d_i}{d_o}$$

Step 2: Insert the known values for the input and output distances ($d_i = 1.6$ ft; $d_o = 3$ ft). Then solve for MA:

$$MA = \frac{d_i}{d_o} = \frac{1.6}{3} = 0.53$$

(rounded)

Step 3: Our calculation indicates that the lever has a mechanical advantage of 0.53, confirming the result from the equation based on force.

You Do the Math

Apply your knowledge to the following questions. Answers are on p. 63.

4. Consider the first-class lever illustrated in Figure 10 (p. 55). Since the lever is balanced, the magnitudes of F_i and F_o equal the magnitudes of f_1 and f_2, respectively. Also, d_i and d_o are equivalent to d_1 and d_2. Answer the following:

a. Calculate the value of mechanical advantage (MA) based on the forces F_i and F_o.

b. Calculate the value of mechanical advantage (MA) based on the distances d_i and d_o.

5. Consider the second-class lever illustrated in Figure 11 (p. 55). Since the lever is balanced, the magnitudes of F_i and F_o equal the magnitudes of f_1 and f_2, respectively. Also, d_i and d_o are equivalent to d_1 and d_2. Answer the following:

a. Calculate the value of mechanical advantage (MA) based on the forces F_i and F_o.

b. Calculate the value of mechanical advantage (MA) based on the distances d_i and d_o.

6. Consider the third-class lever illustrated in Figure 12 (p. 55). Since the lever is balanced, the magnitudes of F_i and F_o equal the magnitudes of f_1 and f_2, respectively. Also, d_i and d_o are equivalent to d_1 and d_2. Answer the following:

a. Calculate the value of mechanical advantage (MA) based on the forces F_i and F_o.

b. Calculate the value of mechanical advantage (MA) based on the distances d_i and d_o.

Summary & Limitations

In these two articles, we have explored basic concepts of leverage. We began in the first installment by considering how out-of-control levers may cause serious injuries in the workplace. We also calculated torque as the product of force times distance. In this installment, we examined the characteristics of three classes of levers and calculated mechanical advantage as the lever's amplification of force. Keep in mind that our calculations assumed friction and the weights of the lever arms were negligible. Our calculations also assumed forces were applied perpendicularly to the lever arms and the levers were balanced. Additional variables must be included in the equations when these assumptions are not met.

FIGURE 14
HOW MUCH HAVE I LEARNED, PROBLEM 7

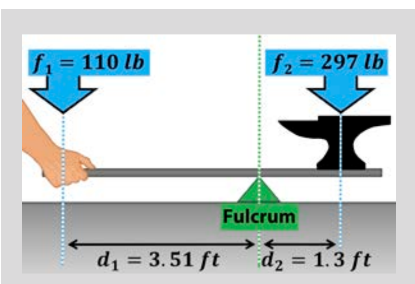


FIGURE 15
HOW MUCH HAVE I LEARNED, PROBLEM 8

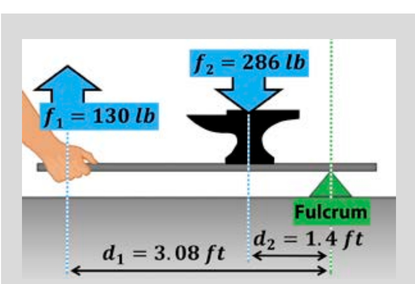
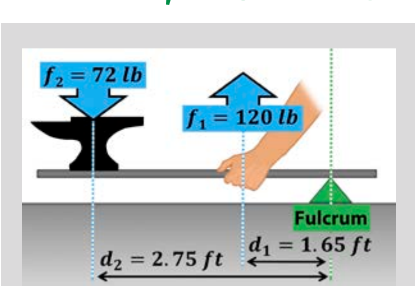


FIGURE 16
HOW MUCH HAVE I LEARNED, PROBLEM 9



How Much Have I Learned?

Try these problems on your own. Answers are on p. 63.

7. Imagine you have used a first-class lever to raise a load consisting of a 297-lb weight, as illustrated in Figure 14. The load's center of gravity is located 1.3 ft to the right of the fulcrum. To keep the load balanced (so it moves neither up nor down), you exert a downward force of 110 lb with your hand. Your hand exerts this force at an average distance of 3.51 ft to the left of the fulcrum. Answer the following questions, based on the assumption that friction and the weights of the lever arms are negligible:

a. How many foot-pounds of torque are created by the effort arm to balance the torque created by the load arm? Use the equation for torque of the effort arm ($|\tau_{effort}|$) and solve in units of foot-pounds (ft-lb).

b. How many foot-pounds of torque are created by the load arm? Use the equation for torque of the load arm ($|\tau_{load}|$) and solve in units of foot-pounds (ft-lb).

c. Calculate the value of mechanical advantage (MA) based on the forces F_i and F_o. Since the lever is balanced, the magnitudes of F_i and F_o equal the magnitudes of f_1 and f_2, respectively.

d. Calculate the value of mechanical advantage (MA) based on the distances d_i and d_o, which are equivalent to d_1 and d_2, respectively.

8. Imagine you have used a second-class lever to raise a load consisting of a 286-lb weight, as illustrated in Figure 15. The load's center of gravity is located 1.4 ft to the left of the fulcrum. To keep the load balanced (so it moves neither up nor down), you exert an upward force of 130 lb with your hand. Your hand exerts this force at an average distance of 3.08 ft to the left of the fulcrum. Answer the following questions, based on the assumption that friction and the weights of the lever arms are negligible:

a. How many foot-pounds of torque are created by the effort arm to balance the torque created by the load arm? Use the equation for torque of the effort arm ($|\tau_{effort}|$) and solve in units of foot-pounds (ft-lb).

b. How many foot-pounds of torque are created by the load arm? Use the equation for torque of the load arm ($|\tau_{load}|$) and solve in units of foot-pounds (ft-lb).

c. Calculate the value of mechanical advantage (MA) based on the forces F_i and F_o. Since the lever is balanced, the magnitudes of F_i and F_o equal the magnitudes of f_1 and f_2, respectively.

d. Calculate the value of mechanical advantage (MA) based on the distances d_i and d_o, which are equivalent to d_1 and d_2, respectively.

9. Imagine you have used a third-class lever to raise a load consisting of a 72-lb weight, as illustrated in Figure 16. The load's center of gravity is located 2.75 ft to the left of the fulcrum. To keep the load balanced (so it moves neither up nor down), you exert an upward force of 120 lb with your hand. Your hand exerts this force at an average distance of 1.65 ft to the left of the fulcrum. Answer the following questions, based on the assumption that friction and the weights of the lever arms are negligible:

a. How many foot-pounds of torque are created by the effort arm to balance the torque created by the load arm? Use the equation for torque of the effort arm ($|\tau_{effort}|$) and solve in units of foot-pounds (ft-lb).

b. How many foot-pounds of torque are created by the load arm? Use the equation for torque of the load arm ($|\tau_{load}|$) and solve in units of foot-pounds (ft-lb).

c. Calculate the value of mechanical advantage (MA) based on the forces F_i and F_o. Since the lever is balanced, the magnitudes of F_i and F_o equal the magnitudes of f_1 and f_2, respectively.

d. Calculate the value of mechanical advantage (MA) based on the distances d_i and d_o, which are equivalent to d_1 and d_2, respectively.

For Further Study

Learn more from the following source: *ASSP's ASP Examination Prep: Program Review and Exam Preparation*, edited by Joel M. Haight, 2016. **PSJ**

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board. The main switch was off, but a switch disconnects power on the load side of the equipment, not the line side. The incoming busses supplying power to the equipment from the utility were still live. The electric company had not been called to shut down utility power. The man, who was in his 20s, suffered a catastrophic burn injury, and was disabled and disfigured. A description of what the man physically had to endure made his deposition difficult to read, but I was most chilled by the emotional pain in his cry that no woman would ever want him. In the other incident, an apprentice died from severe burn injuries. The apprentice was working with a journeyman electrician on live equipment without PPE when an arc flash occurred, which severely injured them both.

I tend to think of wolves disrupting a safe work environment for others. An example is a supervisor who does not wear the required PPE. I have too often seen the behavior of lone wolves result in needless electrical injury. One electrician was fatally shocked by high-voltage test equipment. Three glaring safety violations were committed: 1. He was not wearing the appropriate gloves; 2. he was in contact with the equipment under test; and 3. neither he (by choice) nor the helper were operating a safety switch. If any of these measures had been followed, the electrocution would not have occurred. He and the team leader had about 30 years of experience each and the helper had 10 years' experience. In another incident, two electrical workers were investigating power loss to an industrial process and an arc flash occurred. The severely burned worker was wearing a synthetic-fabric jacket with facility-issued PPE.

Workplace injuries might be reduced if we could transcend the sheep-sheepdog-wolf model and train workers to have the mindset of safety warriors. Some occupations are safer, but the risks of many common occupations are not fully realized. For example, electrical work is often perceived as a dangerous occupation; electrical and nonelectrical workers suffer electrical injuries. However, in 2016, the fatal injury rate for professional drivers was 24.9 (per 100,000 full-time equivalent workers), while the fatal injury rate of electricians was 10. The 2016 Bureau of Labor Statistics data also show that the likelihood of injury was 2.2 times higher for heavy-duty truckers (306) than for electricians (142) and the average number of missed workdays was 2.3 times higher for truck driver injuries (Gammon et al., 2019).

For electrical work, driving and many common jobs, known hazards and hidden perils exist in work environments that seem safe. In sheeplike fashion, workers are trained to recognize typical hazards in their respective workplaces. Yet, workers also need to be trained like warriors to remain alert and vigilant about known and as-yet-unidentified dangers. Equally important, warriors protect and aid others. In too many incidents I have investigated, a worker was injured in the presence of a coworker who was aware of the unsafe work practice but did not intervene. **PSJ**

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Math Toolbox, continued from pp. 52-58

Answers: Understanding Levers

You Do the Math

Your answers may vary slightly due to rounding.

1a. $|\tau_{effort}| = f_1 \cdot d_1 = 50 \cdot 4 = 200\ ft - lb$

1b. $|\tau_{load}| = f_2 \cdot d_2 = 100 \cdot 2 = 200\ ft - lb$

2a. $|\tau_{effort}| = f_1 \cdot d_1 = 60 \cdot 2.5 = 150\ ft - lb$

2b. $|\tau_{load}| = f_2 \cdot d_2 = 300 \cdot 0.5 = 150\ ft - lb$

3a. $|\tau_{effort}| = f_1 \cdot d_1 = 155 \cdot 1.24 = 192.2\ ft - lb$

3b. $|\tau_{load}| = f_2 \cdot d_2 = 62 \cdot 3.1 = 192.2\ ft - lb$

4a. $MA = \frac{F_o}{F_i} = \frac{100}{50} = 2$

4b. $MA = \frac{d_i}{d_o} = \frac{4}{2} = 2$

5a. $MA = \frac{F_o}{F_i} = \frac{300}{60} = 5$

5b. $MA = \frac{d_i}{d_o} = \frac{2.5}{0.5} = 5$

6a. $MA = \frac{F_o}{F_i} = \frac{62}{155} = 0.40$

6b. $MA = \frac{d_i}{d_o} = \frac{1.24}{3.1} = 0.40$

How Much Have I Learned?

7a. $|\tau_{effort}| = f_1 \cdot d_1 = 110 \cdot 3.51 = 386.1\ ft - lb$

7b. $|\tau_{load}| = f_2 \cdot d_2 = 297 \cdot 1.3 = 386.1\ ft - lb$

7c. $MA = \frac{F_o}{F_i} = \frac{297}{110} = 2.7$

7d. $MA = \frac{d_i}{d_o} = \frac{3.51}{1.3} = 2.7$

8a. $|\tau_{effort}| = f_1 \cdot d_1 = 130 \cdot 3.08 = 400.4\ ft - lb$

8b. $|\tau_{load}| = f_2 \cdot d_2 = 286 \cdot 1.4 = 400.4\ ft - lb$

8c. $MA = \frac{F_o}{F_i} = \frac{286}{130} = 2.2$

8d. $MA = \frac{d_i}{d_o} = \frac{3.08}{1.4} = 2.2$

9a. $|\tau_{effort}| = f_1 \cdot d_1 = 120 \cdot 1.65 = 198\ ft - lb$

9b. $|\tau_{load}| = f_2 \cdot d_2 = 72 \cdot 2.75 = 198\ ft - lb$

9c. $MA = \frac{F_o}{F_i} = \frac{72}{120} = 0.6$

9d. $MA = \frac{d_i}{d_o} = \frac{1.65}{2.75} = 0.6$